Chap1

2023年3月7日 18:04

1. 略

3. 略

4. $P(A+B+C)=P(A)+P(B)+P(C)$
$-P(AB)-P(AC)-P(BC)$
$+P(ABC)$

5. (1) $P(\text{只甲})=P(\text{甲})-P(\text{甲乙})=25\%$

(2) $P(\text{只乙})=P(\text{乙})-P(\text{甲乙})=10\%$

$P(\text{只一})=25\%+10\%=35\%$

(3) $P(\text{至少一})=1-P(\text{只一})-P(\text{甲乙})$
$=1-35\%-15\%$
$=50\%$

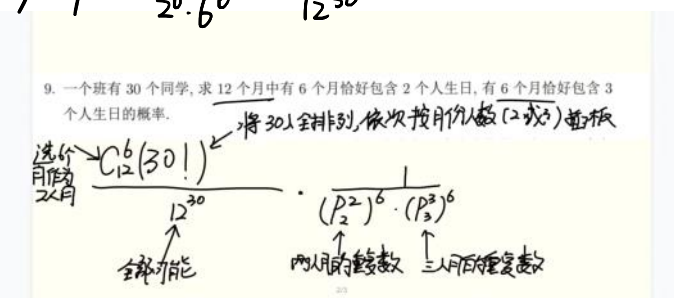
(4) $P(\text{都不})=1-50\%=50\%$

6. (1) $A\subseteq B$. $P(AB)=P(A)=0$

(2) $A\cdot B$ 独立. $P(AB)=0$

7. $P(AB)\in[0,\min(a,b)]$

9. $P=\dfrac{30!}{2^6\cdot 6^6}\cdot\dfrac{C_{12}^{6}}{12^{30}}$

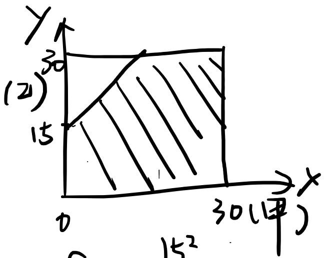

9. 一个班有 30 个同学, 求 12 个月中有 6 个月恰好包含 2 个人生日, 有 6 个月恰好包含 3 个人生日的概率.

10. 4个空位连着的概率：$\dfrac{C_9^1}{C_{12}^8}$
≈ 0.018，是小概率事件.

所以应该是人为的.

11、?

18.
(2)



$$P=\frac{900-\frac{15^2}{2}}{900}=87.5\%$$

21. (1) 有实根 $\Delta=B^2-4C\geq 0 \Rightarrow A$

$A=\{(2,1),(3,1),(4,1),(5,1),(6,1),$
$(3,2),(4,2),(5,2),(6,2)$
$(4,3),(5,3),(6,3),$
$(4,4),(5,4),(6,4),$
$(5,5),(6,5)$
$(6,6)\}$

$P(A)=\frac{1}{2}$

(2) 有重根: B

$B=\{(2,1),(4,4)\}$

$P(B)=\frac{1}{18}$

22. (1) $P=\frac{A_{10}^8}{10^8}=3.1\%$

(2) $P=\frac{10}{10^8}=0.000001\%$

(3) $P=\frac{C_8^3\cdot 9^5}{10^8}=3.31\%$

24. (1) 设A: 取一等品. B_1: 取第一箱. B_2: 取第二箱

$P(A)=P(A|B_1)P(B_1)+P(A|B_2)P(B_2)$

$$\frac{1}{5}\times\frac{1}{2}+\frac{3}{5}\times\frac{1}{2}=\frac{2}{5}$$

(2) 设C：第二次也取一等品

$$P(C|A)=\frac{P(AC)}{P(A)}=\frac{P(AC|B_1)P(B_1)+P(AC|B_2)P(B_2)}{P(A)}$$

$$=\frac{\frac{1}{2}\times\frac{1}{5}\times\frac{9}{49}+\frac{1}{2}\times\frac{3}{5}\times\frac{17}{29}}{\frac{2}{5}}$$

$$=48.56\%$$

25. $P=\frac{r}{r+b}\times\frac{r+a}{r+b+a}\times\frac{b}{r+b+2a}\times\frac{b+a}{r+b+3a}$

26. B_1：一号车间 B_2：二号车间 B_3：三号车间

A：抽到次品

(1) $P(A)=P(A|B_1)P(B_1)+P(A|B_2)P(B_2)+P(A|B_3)P(B_3)$

$=1\%\times\frac{1}{2}+1\%\times\frac{1}{3}+2\%\times\frac{1}{6}$

$=1.17\%$

(2)

$$P(B_1|A)=\frac{P(B_1A)}{P(A)}=\frac{P(A|B_1)P(B_1)}{1.17\%}$$

$$=\frac{1\%\times\frac{1}{2}}{1.17\%}=42.74\%$$

27. A：男性

B：色盲

$$P(A|B)=\frac{P(AB)}{P(B)}=\frac{P(B|A)P(A)}{P(B|A)P(A)+P(B|\bar{A})P(\bar{A})}$$

$$= \frac{0.05 \times \frac{1}{2}}{0.05 \times \frac{1}{2} + 0.0025 \times \frac{1}{2}}$$

$$= 95.24\%$$

28. (1) A_1, A_2, A_3: 三个地区

B: 第一次抽女生 C: 第二次抽男生

$$P(B) = \frac{3+7+5}{50} = 34\%$$

(2) $P(B|C)$

$$= \frac{P(BC)}{P(C)} = \frac{\sum_{i=1}^{3} P(BC|A_i)P(A_i)}{\sum_{i=1}^{3} P(BC|A_i)P(A_i) + \sum_{i=1}^{3} P(\overline{B}C|A_i)P(A_i)}$$

$$= \frac{\frac{1}{5} \times \frac{3}{10} \times \frac{7}{9} + \frac{3}{10} \times \frac{7}{15} \times \frac{8}{14} + \frac{1}{2} \times \frac{5}{25} \times \frac{20}{24}}{\sim + \frac{1}{5} \times \frac{7}{10} \times \frac{6}{9} + \frac{3}{10} \times \frac{8}{15} \times \frac{7}{14} + \frac{1}{2} \times \frac{20}{25} \times \frac{19}{24}}$$

$$= \frac{0.21}{0.21 + 0.49} = 30\%$$

29 A_i: 扔了 i 个红球 B: 摸红

$$P(A_0) = \frac{C_5^5}{C_{25}^5} = 0.00188\% \qquad P(B|A_0) = 100\%$$

$$P(A_1) = \frac{C_{20}^1 C_5^4}{C_{25}^5} = 0.188\% \qquad P(B|A_1) = \frac{19}{20} = 95\%$$

$$P(A_2) = \frac{C_{20}^2 C_5^3}{C_{25}^5} = 3.58\% \qquad P(B|A_2) = \frac{18}{20} = 90\%$$

$$P(A_3) = \frac{C_{20}^3 C_5^2}{C_{25}^5} = 21.46\% \qquad P(B|A_3) = \frac{17}{20} = 85\%$$

$$P(A_4) = C_{20}^4 C_5^1 = 45.6\% \qquad P(B|A_4) = \frac{16}{20} = 80\%$$

$$P(A_4)=\frac{C_{20}^4C_5^1}{C_{25}^5}=45.6\% \qquad P(B|A_4)=\frac{16}{20}=80\%$$

$$P(A_5)=\frac{C_{20}^5}{C_{25}^5}=29.18\% \qquad P(B|A_5)=\frac{15}{20}=75\%$$

$$P=\frac{\sum_{i=2}^{2}P(A_i)P(B|A_i)}{\sum_{i=0}^{5}P(A_i)P(B|A_i)}$$

$$=99.77\%$$

30.(1) A：阳性　B：带菌

$$P(B|A)=\frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|\bar{B})P(\bar{B})}$$

$$=\frac{0.95\times0.1}{0.95\times0.1+0.01\times0.9}$$

$$=91.3\%$$

(2) C：两次阳

$$P(B|C)=\frac{P(C|B)P(B)}{P(C|B)P(B)+P(C|\bar{B})P(\bar{B})}=\frac{0.95^2\times0.1}{0.95^2\times0.1+0.01^2\times0.9}$$

$$=99.9\%$$

31、A：取红　B_i：取第i个笔筒

(1) $P(A)=\frac{2+4+3}{6+6+6}=\frac{1}{2}$

(2) $P(B_1|A)=\frac{P(B_1A)}{P(A)}=\frac{\frac{2}{18}}{\frac{1}{2}}=\frac{4}{18}$

$P(B_2|A)=\frac{P(B_2A)}{P(A)}=\frac{\frac{4}{18}}{\frac{1}{2}}=\frac{4}{9}$

$P(B_3|A)=\frac{P(B_3A)}{P(A)}=\frac{\frac{3}{18}}{\frac{1}{2}}=\frac{1}{3}$

第2个取出的概率大

32. A：收0　B：真实为0

$P(B|A)=\frac{P(A|B)P(B)}{P(A|B)P(B)+P(A|\bar{B})P(\bar{B})}$

$=\frac{0.98\times\frac{2}{3}}{0.98\times\frac{2}{3}+0.01\times\frac{1}{3}}=99.49\%$

33. A_i：甲中放3i个白球到乙

B：从乙摸白

$P(A_0)=\frac{C_2^2}{C_7^2}=\frac{1}{21}$　$P(B|A_0)=\frac{4}{11}$

$P(A_1)=\frac{C_5^1C_2^1}{C_7^2}=\frac{10}{21}$　$P(B|A_1)=\frac{5}{11}$

$P(A_2)=\frac{C_5^2}{C_7^2}=\frac{10}{21}$　$P(B|A_2)=\frac{6}{11}$

(1) $P(B)=\sum_{i=0}^{2}P(A_i)P(B|A_i)=\frac{38}{77}\approx 49.36\%$

(2) $P=1-P(A_0|B)=1-\frac{P(B|A_0)P(A_0)}{P(B)}=1-\frac{\frac{1}{21}\times\frac{4}{11}}{\frac{38}{…}}$

(2) $P = 1 - P(A_0|B) = 1 - \dfrac{P(B|A_0)P(A_0)}{P(B)} = 1 - \dfrac{4\times 11}{\frac{38}{77}}$

$= \dfrac{37}{38} \approx 97.37\%$

34. $P(B|A) = \dfrac{P(AB)}{P(A)}$

$P(B|\bar{A}) = \dfrac{P(B\bar{A})}{P(\bar{A})} = \dfrac{P(B) - P(AB)}{1 - P(A)}$

$\therefore \dfrac{P(AB)}{P(A)} = \dfrac{P(B) - P(AB)}{1 - P(A)}$

$\Rightarrow P(AB) = P(A)P(B)$

$\therefore A, B$ 独立

35. $P(B|A) = \dfrac{P(AB)}{P(A)} > P(B)$

$\dfrac{P(A) - P(A\bar{B})}{P(A)} > 1 - P(\bar{B})$

$1 - \dfrac{P(A\bar{B})}{P(A)} > -P(\bar{B})$

$\dfrac{P(A\bar{B})}{P(\bar{B})} < P(A)$

$\dfrac{P(\bar{B}) - P(\bar{A}\bar{B})}{P(\bar{B})} < 1 - P(\bar{A})$

$P(\bar{A}\bar{B})$ $P(\bar{A})$

$$\frac{\cdots}{P(\overline{B})} > P(A)$$

$$\frac{P(\overline{A}\overline{B})}{P(\overline{A})} > P(\overline{B})$$

$$P(\overline{B}|\overline{A}) > P(\overline{B})$$

$\therefore \overline{A}$ 倾向于 $\overline{B}$

36. $P(\overline{A}\overline{B}) = 1-0.12 = 0.88$

$P(AB) = 0.1 \quad P(A\overline{B}+\overline{A}B) = 0.02$

设 $P(A\overline{B}) = x$, $P(\overline{A}B) = 0.02-x$

$P(A) = P(AB)+P(A\overline{B}) = 0.1+x$

$P(B) = 0.12-x$

若 A,B 独立, $P(AB) = P(A)P(B)$

$$0.1 = (0.1+x)(0.12-x)$$

$$x^2 - 0.02x + 0.088 = 0$$

$\Delta < 0$ 无解 矛盾

故 A,B 不独立

37. $P(AB|C) = P(A|C)P(B|C) = 0.81$

$P(AB|\overline{C}) = P(A|\overline{C})P(B|\overline{C}) = 0.02$

$P(AB) = P(AB|C)P(C) + P(AB|\overline{C})P(\overline{C})$

$= 0.81\times0.5 + 0.02\times0.5$

$= 0.415$

$P(A)=P(A|C)P(C)+P(A|\bar{C})P(\bar{C})$

$=0.9\times0.5+0.2\times0.5$

$=0.55$

$P(B)=P(B|C)P(C)+P(B|\bar{C})P(\bar{C})$

$=0.9\times0.5+0.1\times0.5$

$=0.5$

$P(AB)\neq P(A)P(B)$ ∴ A、B 不相互独立

38、(1) $P=0.5\times0.4\times0.2+0.5\times0.6\times0.2+0.5\times0.4\times0.8$

$=0.26$

(2) $P=1-0.5\times0.4\times0.2$

$=0.96$

39.(1) $P_AP_BP_C$

(2) $1-(1-P_A)(1-P_B)(1-P_C)$

(3) $1-(1-P_A^2)(1-P_B^2)(1-P_C^2)$

(4) $P_D^2[1-(1-P_A)(1-P_B)(1-P_C)]$

(5) 摆

40. $P=1-[0.7\times(1-0.4\times0.6)]$

$=0.468$